Non commutative quantum spacetime with topological vortex states,
and dark matter in the Universe

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Abstract

Non commutative geometry is creating new possibilities for physics. Quantum spacetime geometry and post inflationary models of the universe with matter creation have an enormous range of scales of time, distances and energy in between. There is a variety of physics possible till the nucleosynthesis epoch is reached. The use of topology and non commutative geometry in cosmology is a recent approach. This paper considers the possibility of topological solutions of a vortex kind given by non commutative structures. These are interpreted as dark matter, with the grand unified Yang-Mills field theory energy scale used to describe its properties. The relation of the model with other existing theories is discussed.

Introduction

The role of Geometry and Topology in understanding the physics of the universe of space time and matter has a century of work that is supported by observations. Inflationary cosmological models developed for creation of matter have included the small anisotropic distributions as given by COBE and WMAP observations. The models modified to create the voids and filamentary large scale structure of the early universe , with the ratios of dark energy, dark matter and normal matter are topics of active research.

The non linear evolution equations of the early universe give rise to anisotropy and inhomogenity persisting at all length scales , these are observable with even finer resolution that is expected in subsequent experiments. These indicate analogies with condensed matter. Topological defects and vortex like states in rapid expansion and cooling occur in condensed matter phases.

This paper develops the role of noncommutative geometry in the formation of the vortex like topological states. The typical model of the universe, a few seconds old and a few kilometers in size, is expected to have some 10^{80} particles of normal matter, and to form a smooth geometry given by Einstein's theory of general relativity ; with 10^{-5} parts fluctuation in matter distribution as inferred from the temperature and thermal radiation microwave measurements. [References].

The models of the early universe

In the inflation models and their variations that give the later stage expanding universe models with the correct observed properties, there are topologically non trivial states possible. A rubber sheet or expanding balloon

picture with locally ' pinched and popped ' regions is suggested. These topological 'defects'could be candidates for dark matter with a long (billion years) half life for decay into normal matter. A topological defect would give rise to a graviton and a photon as it 'pops out of the pinch', and gives a smooth manifold in its neighbourhood. Typical masses of 10^{-15} kilograms have been suggested, which would give with ($E = mc^2$) ,gamma rays of 10^{21} electron volts ; generating normal matter and the graviton emitted would propagate as a gravitational wave contributing to the smoothing out of the 'wrinkles' in local space time. From a $1 : 100$ to a $1 : 30$ ratio of normal to dark matter are reported in the early universe and a ratio $1 : 8$ in galaxies and their halos. The slow decay could be a perturbative or non perturbative effect; but it involves a change in topology. Ultra heavy dark matter at the 10^{15} Gev GUT scale and its decay into gamma rays and baryon ,lepton production are known as an alternative to the baryogenesis stage of the universe.

The decaying topological defects give rise to the 10^{21} e.v. gamma ray emission consistent with cosmic rays of highest energy gamma rays observed, and the nearly isotropic distribution of the gamma ray bursts in the early Universe. The dark matter in galaxy halos is the remnant of the earlier rapidly decaying stage ,possibly an exponential decrement of the number of the topological defects. The grand unification (GUT) scale of 10^{15} Gev , is also comparable and indicates that a single Holonomy gauge field , with gauge bosons treated as nearly massless, and represented by the single coupling constant would describe the loop integral around the ' defect' as it decays. In theories of Quantum chromodynamics of the quark gluon plasma phase transitions and the inflation cosmological models with a scalar field , topological defects are possible. At a scale between GUT (10^{15} Gev) and of Quantum Riemann geometry (10^{19} Gev), a non commutative structure can arise in the non linear regime of the evolution of the combined model of all the dynamics of the universe. The so called vaccuum manifold $M-> G/H$ as a symmetry breaking with group G and subgroup H gives rise to topologically non trivial solutions.

The non commutative θ structure with a length scale of 10^{-27} meters ; h/mc with $m = 10^{-15}$ kg, for the vortex like topological defect , goes over to a commutative differential geometry in the large scale. It is possible to retain the symmetric metric and connection and absence of torsion in the large scale. In a geometric interpretation of charge the continuous electromagnetic field F has a topological index as an integral of F*F , as also for gravity of geons in geometrodynamics. The topological defects of dark matter arise in the background geometry due to the non commutative structure with a 10^{-27} length scale. The number and number density, mass and mass density of the dark matter as topological states in the universe are still not known.

As these 'topological defect' regions are excluded from the else where smooth geometry, they do not interact with electromagnetic radiation and are 'dark' matter; better called 'dark' space time matter.

At the Planck scale, theories of Quantum Gravity such as loop space quantum gravity and quantum Riemann geometry give a spin network representation with area and volume operators and holonomy for the fields. Quantum space time with a non commutative geometry is proposed as an alternative to theories with other forms of exotic matter , primordial black holes and sources of gamma bursts. The presence of vorticity can be described by a general Stokes theorem and the second exterior derivative . In the condition for exact and closed forms, the topology of the manifold plays a role; the genus number , betti numbers, Euler numbers, Hopf and Pontryagin index and other topological indexes. The non inertial vaccuum in the accelerated expanding geometry gives rise to particle creation by Bogoliubov transforms. Topologically non trivial solutions are known for a variety of non linear evolution equations. The quartic potential scalar field with a non zero vaccuum state in the standard inflation models is the starting point for various theories of early cosmology. The field could be a residue from the transition of non commutative to commutative geometry and thus have a geometrised rather than independent existence.

The Model including non commutative geometry

Consider the vector field χ that has the non commutative structure $\chi_i\chi_j - \chi_j\chi_i = i\theta_{ij}l^2$.

The spacetime coordinates are obtained as $\chi_i - > x_i$ in the limit of commutative geometry.

The congugate momenta have the relation $p_ip_j - p_jp_i = \theta_{ij}^{-1}l^{-2}h^2/4\pi^2$.

Therefore the usual quantisation rule of $\chi_ip_j - p_i\chi_j = ih/2\pi\delta_{ij}$.

This makes consistent the non commutative and symplectic structures. The SU(2) version of the gravity gauge theory expressed as a holonomy in loop space is possible to construct in the commutative geometry limit. The field χ is chosen to have the exact and closed form conditions that fail at the vortex like topological defects. The length scale for the non commutative structure is chosen small enough for the spacetime to be normal almost everywhere to do the usual physics. Are topologically non trivial states consistent with the standard general relativity using commutative geometry and is the use of non commutative structures the only way , or one of the ways, for them to occur is a unresolved question. The consistency of the theory of Yang-Mills type of all fields with non commutative geometry is being established from its inception.There are likely to be connections between loop space quantum gravity and quantum Riemannian geometry with non commutative geometry.

The characterisation of the topological states arising in the cosmological models is done with conditions on the vector field introduced for non commutative geometry. In differential geometry usually the Stokes law would read $\int_M d\omega = \int_{\partial M} \omega$, and $d^2\chi = 0$ every where except at the topological defects. The χ field as a vortex like field will have $d^2\chi \neq 0$; $\omega = d\chi$. This fails at the topological defects. The non commutative structure suggests that these defects are vortex like. The vortex has a 'length' scale given by l, and the θ is a skew symmetric form or matrix, so diagonal terms are zero. If the other terms also equal zero then those fields or coordinates commute. The planar vortex for example is described as two non commuting variables, with a θ_{12} form, such that the exterior derivative d^2 is non zero with an exclusion at the origin, such as $(\chi_1 d\chi_2 - \chi_2 d\chi_1) * ((\chi_1)^2 + (\chi_2)^2)^{-1}$. In usual vector calculus the curl of the gradient of a scalar field and the divergence of the curl of a vector field are zero. Or as expressed in terms of differential forms the d^2 is zero. The de Rham integration on non simply connected spaces and those with homotopy and cohomology , gives the condition for the failure of these differential and integral conditions. This indicates presence of vorticity and topologically non trivial states or 'defects'.

Consider a simple example of a holonomy loop of the U(1) group around the vortex like topological defect : $Trace P exp(-\oint A dx)$. That is the 'pinched and popped' defect in space time which is the dark matter candidate. The length scale of 10^{-27} meters for the loop is associated with the likely mass of 10^{-15}kg ; and the gauge field A is the massless multiplet of GUT gauge fields at 10^{15} Gev. The homotopy group $\pi_1(S^1) = Z$ and $\pi_1(SU(N)) = Z$. Higher homotopy groups for higher dimension spheres could be considered. The fundamental group gives the set of integers and a winding number interpretation. A suitable Hopf index can be obtained from the $\oint \omega \wedge d\omega$ integral. This is a topological invariant on the 3 spheres in space time. The confinement of 10^{-15} kilograms to a region of size 10^{-27} meters in a topologically 'excluded' part of the spacetime manifold, does not create collapse to a black hole as the Schwarzschild radius for that would be around 10^{-40} meters. This mass energy can be released into the surrounding normal commutative spacetime when the defect decays quantum theoretically into graviton and ultra relativistic (GUT scale) effectively massless gauge bosons. In the holonomy representation these are represented by a U(1) gauge photon, as a non commutative vortex state is destroyed for which the fundamental group was Z, the group of integers.

The abundance of dark matter expected in the universe implies that these 'defects' occur abundantly, and that the tiny length scales of the loops around them means that for the most part smooth geometry prevails and general coordinate transformations required for Einstein's Theory of general relativ-

ity, remain valid everywhere except in these topologically excluded regions in which most mass/energy resides. The nature of dark energy is less certain and could be the cosmological term in Einstein's theory and hence at length scales much bigger than the Planck scale or the noncommutative vortex scales. Another major question is the role of the scalar field that is used in inflation models and the Higgs scalar field used in particle physics in these topological solutions. Will it be neccessary to have additional fields to describe dark matter and energy , or are they already contained in the theory, with solutions including the non commutative structure. The reheating phase after the rapid cooling phase in inflation models may not destroy the topological states , although it is supposed to smooth out other anisotropies and inhomogenities.

Vortex formation is associated with angular momentum and the spin network of the quantum space time foam indeed gives the quantised geometric operators of area nad volume in Quantum Riemannian geometry as for example, area proportional to $(l_p)^2(j(j+1))^{1/2}$. From the Planck scale $l_p = 10^{-35}$ meters to the non commutative scale of $l = 10^{-27}$ meters, the continuum limit is reached ; and derivation ,commutation operations can be defined. This is taken at the upper limit in energy of the GUT scale with the 0.033 asymptotic converging coupling constant . This is because operationally, the probes for finer grain topology of the space time can be only the GUT scale ; as our measurement methods depend on particle states with respect to locally Lorentzian spacetime. If astronomical observations also give the gamma energies corresponding to a decaying topological defect of this scale, then there is a consistency. The defects of walls, strings, monopoles and texture type have been considered in other work and found to have limitations in explaining the dark matter and decay into high energy gamma rays.

Conclusion

The correspondence with other theories could be made for (a) Vortex and topological defect formation in field theory and condensed matter theories, (b) Quantum Riemann geometry and loop space quantum Yang Mills field theory and quantum gravity theories, (c) Non commutative geometry theories for field theory and condensed matter theory,(d) Inflationary and model cosmologies that give the anisotropy and matter creation observed in the universe.

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